DEFORMATION OF THE VELOCITY PROFILE IN AN INHOMOGENEOUS MAGNETIC FIELD

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When a conductive medium flows along a channel in an inhomogeneous external magnetic field, the associated electrical current and velocity are likewise inhomogeneous. The electric field in such channels has been computed on several occasions [1 and 2] under the assumption of a nondeformable velocity profile (valid at a small magnetohydrodynamic interaction parameter). The resulting distributions of the electromagnetic parameters can be used to find the fields of the hydrodynamic quantities in the first approximation . This approach was used by Shurcliff [3] to determine the asymptotic velocity profile established in the stream following its passage through the inhomogeneous magnetic field zone in a channel with nonconductive walls. The flow in a channel with electrodes in the case of an incompressible fluid is computed in [4] ; analogous computations for an ideal perfect gas are carried out in [5 to 7]. In the aforementioned papers the flow unperturbed by the magnetic field is assumed to be homogeneous and anisotropy of the medium is neglected. The present paper concerns the effect of anisotropic conductivity and initial flow inhomogeneity on the deformation of the velocity profile in an inhomogeneous magnetic field.

1. The flow of an incompressible nonviscous (*) fluid in a flat channel (Fig. 1) $|x^\circ| < \infty$, $0 < y^\circ < h = \text{const}$ in the presence of an external magnetic field $\mathbf{B} = (0, 0, B^*, b(x))$ for small magnetic Reynolds numbers is described by the system

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + sbj_y, \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} - sbj_x \qquad \left(s = \frac{\sigma B^{*2}h}{c^2\rho V}\right) \tag{1.1}$$

$$j_x = \frac{1}{1+\beta^2b^2}\left[-\frac{\partial\varphi}{\partial x} + vb + \beta b\left(ub + \frac{\partial\varphi}{\partial y}\right)\right], \qquad \frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} = 0$$

$$j_y = \frac{1}{1+\beta^2b^2}\left[-\frac{\partial\varphi}{\partial y} - \beta b\left(\frac{\partial\varphi}{\partial x} - vb\right) - ub\right] \qquad \left(\beta = \frac{eB^*\tau}{mc}\right)$$

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*) The extent (in the x° direction) of the inhomogeneous magnetic field usually does not exceed the height h of the channel in order of magnitude. Hence, the effects of viscosity which are manifested over much larger segments of the channel can be neglected in the first approximation.

Here u, v and j_x, j_y are the dimensionless components of the velocity and density vectors of the electric field, respectively, p and φ are the dimensionless pressure and electrical potential, σ is the electrical conductivity, ρ is the density, B^* is the characteristic value of the magnetic induction, V is the average velocity over the channel cross section, c is the velocity of light in a vacuum, s is the magnetohydrodynamic interaction parameter, e and m are the electron charge and mass, τ is the average time between collisions of electrons with other particles, and β is the Hall parameter. The quantities β and σ will henceforth be assumed constant. The velocity, electric current density, pressure, electrical potential, and coordinates are given in ratio to the quantities V, $c^{-1}\sigma VB^*$, ρV^2, $c^{-1}hVB^*$ and h, respectively.

System (1.1) must be supplemented by boundary conditions for the hydrodynamic and electrical quantities and by asymptotic conditions for $|x| \to \infty$. With a small magnetohydrodynamic interaction parameter the solution of system (1.1) can be sought in series form.

$$u = u^-(y) + \sum_{k=1}^{\infty} s^k u_k(x, y), \qquad \varphi = \sum_{k=0}^{\infty} \varphi_k(x, y) s^k$$
$$v = \sum_{k=1}^{\infty} s^k v_k(x, y), \qquad j_x = \sum_{k=0}^{\infty} s^k j_{xk}(x, y) \tag{1.2}$$
$$p = p^- + \sum_{k=1}^{\infty} s^k p_k(x, y), \qquad j_y = \sum_{k=0}^{\infty} s^k j_{yk}(x, y)$$

Here $v^- = 0$, $u^-(y)$, and $p^- = \text{const}$ are the velocity and pressure in the channel for $s = 0$. Substituting (1.2) into (1.1) for each $k = 0, 1, 2, \ldots$, we obtain two linear systems in φ_k, j_{xk}, j_{yk} and u_{k+1}, v_{k+1}, p_{k+1} respectively. The zeroth approximation for the electrical parameters and the first approximation for the hydrodynamic parameters are of the form

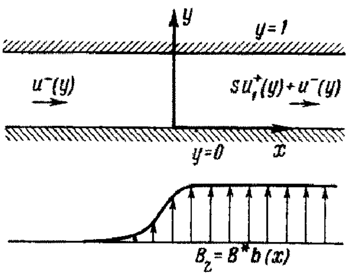

Fig. 1

$$j_{x0} = \frac{1}{1+\beta^2 b^2}\left[-\frac{\partial \varphi_0}{\partial x} + \beta b\left(u^- b + \frac{\partial \varphi_0}{\partial y}\right)\right]$$
$$\frac{\partial j_{x0}}{\partial x} + \frac{\partial j_{y0}}{\partial y} = 0 \tag{1.3}$$
$$j_{y0} = \frac{1}{1+\beta^2 b^2}\left[-\frac{\partial \varphi_0}{\partial y} - u^- b - \beta b \frac{\partial \varphi_0}{\partial x}\right]$$

$$u^- \frac{\partial u_1}{\partial x} + v_1 \frac{du^-}{dy} = -\frac{\partial p_1}{\partial x} + b j_{y0} \tag{1.4}$$
$$u^- \frac{\partial v_1}{\partial x} = -\frac{\partial p_1}{\partial y} - b j_{x0} \qquad \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0$$

System (1.3) with $u^- \equiv 1$ has been investigated by many authors [1 and 2]. System (1.4) for $u^- \equiv 1$ and $\beta = 0$ in Equations (1.3) is considered in [3 and 4].

Let us average Equations (1.4) over the channel cross section. Assuming that the channel walls are impermeable to the fluid, we obtain

$$2\frac{d}{dx}\langle u^- u_1\rangle = -\frac{d}{dx}\langle p_1\rangle + b\langle j_{y0}\rangle, \qquad \langle u_1\rangle = 0$$
$$\frac{d}{dx}\langle u^- v_1\rangle = p_1(x, 0) - p_1(x, 1) - b\langle j_{x0}\rangle \qquad \left(\langle a\rangle = \int_0^1 a\,dy\right) \tag{1.5}$$

The latter condition in (1.5) defines the averaging operation. We further assume that $b(-\infty) = 0$, and that the magnetic field and boundary conditions for the electrical current are homogeneous at infinity on the right. Then

$$u_1(-\infty) = 0,\ p_1(-\infty) = 0,\ \mathbf{j}_0(-\infty) = 0;\ \mathbf{j}_0(\infty) = \text{const},$$

if $b(\infty) \neq 0$ and $\mathbf{j}_0(\infty) = 0$, if $b(\infty) = 0$. The current and potential distribution for $x = \infty$ can be determined from the solution of system (1.3) by setting $b(x)$ = const, $\mathbf{j}_0$ = const. Thus, if $b(\infty) = 1$, then $\mathbf{j}_0(\infty) = 0$ provided the walls are nonconductive for $x \to \infty$; $j_{x0}(\infty) = -\beta j_{y0}(\infty)$, $j_{y0}(\infty) = -(1-K)/(1+\beta^2)$, when the walls are continuous electrodes; $j_{x0}(\infty) = 0$, $j_{y0}(\infty) = -(1-K)$ in the case of ideally segmented electrodes. Here $K = \varphi(\infty, 0) - \varphi(\infty, 1) = \text{const}$ is the parameter of the load connecting the electrodes.

The pressure drop $\Pi(x) = s\rho V^2 P(x)$ in the channel is (by virtue of (1.5)) given by Formula

$$P = 2\langle u^- u_1\rangle - \int_{-\infty}^{x} b\langle j_{y0}\rangle\, dx = \tag{1.6}$$

$$= 2\langle u^- u_1\rangle - xb(\infty)j_{y0}(\infty) - \int_{-\infty}^{0} b\langle j_{y0}\rangle\, dx - \int_{0}^{x}[b\langle j_{y0}\rangle - b(\infty)j_{y0}(\infty)]\, dx$$

$$(P = p_1(-\infty) - \langle p_1(x, y)\rangle = -\langle p_1(x, y)\rangle)$$

For large x (for which the current distribution is homogeneous), Equation (1.6) yields

$$P = P_\infty = 2K^* - xb(\infty)j_{y0}(\infty) - \int_{-\infty}^{0} b\langle j_{y0}\rangle\, dx - \int_{0}^{\infty}[b\langle j_{y0}\rangle - b(\infty)j_{y0}(\infty)]\, dx \tag{1.7}$$

$$(K^* = \langle u^- u_1^+\rangle,\ u_1^+ = u_1(\infty, y))$$

If $j_{y0}(\infty) = 0$, then

$$P_\infty = 2K^* - \int_{-\infty}^{+\infty} b\langle j_{y0}\rangle\, dx \tag{1.8}$$

For $u^- \equiv 1$ we have $K^* = 0$, and the pressure drop can be determined without the solution of system (1.4). If, on the other hand, $u^- \not\equiv 1$, then in order to compute P_∞ it is necessary not only to solve (1.4), but also to find $u_1^+(y)$ on the basis of Equations (1.4).

Shurcliff [3] showed that for $u^- \equiv 1$ the asymptotic velocity profile $u_1^+(y)$ can be computed without solving the whole of system (1.4). We shall show that this conclusion is also valid for $u^- \not\equiv 1$. Constructing the difference between the first two equations of system (1.4) differentiated with respect to y and x, respectively, and making use of the continuity equations for the velocity and electrical current density, we obtain

$$u^- \frac{\partial^2 u_1}{\partial x\,\partial y} - \frac{d^2 u^-}{dy^2}\frac{\partial}{\partial x}\int_0^y u_1 dy - u^- \frac{\partial^2 v_1}{\partial x^2} = j_{x0}\frac{db}{dx} \tag{1.9}$$

Integrating this equation with respect to x over the limits $(-\infty, +\infty)$ and taking account of the fact that $u_1 \to u_1^+(y)$, $v_1 \to 0$ as $x \to \infty$, we obtain the following ordinary second-order differential equation:

$$u^- \frac{d^2\phi}{dy^2} - \frac{d^2u^-}{dy^2}\phi = \gamma(y), \qquad \phi(0) = 0, \qquad \phi(1) = 0 \qquad (1.10)$$

$$\left(\phi(y) = \int_0^y u_1^+ dy, \quad \gamma(y) = \int_{-\infty}^{\infty} j_{x0} \frac{db}{dx}\, dx\right)$$

If $b(x)$ = const everywhere in the channel, then by (1.10) there is no velocity perturbation. In regions where $b(x)$ = const the electromagnetic force is potential, so that the vorticity is conserved along the current lines. As $x \to \infty$ the electromagnetic force is completely balanced by the pressure gradient. Deformation of the velocity profile is due to the existence of segments with an inhomogeneous magnetic field.

The solution ϕ of Equation (1.10) and the velocity correlation K^* are given (*) by Formulas

$$\phi = Cu^-\zeta(y) + u^-\xi(y), \qquad K^* = \langle u_1^+ u^- \rangle = \frac{1}{2\zeta(1)}\int_0^1 \gamma(y)\,\psi(y)\,dy$$

$$u_1^+(y) = C\left(\frac{1}{u^-} + \zeta(y)\frac{du^-}{dy}\right) + \frac{\delta(y)}{u^-} + \xi(y)\frac{du^-}{dy}, \qquad C = -\frac{\xi(1)}{\zeta(1)} \qquad (1.11)$$

$$\left(\zeta(y) = \int_0^y \frac{dy}{u^{-2}}, \quad \delta(y) = \int_0^y \gamma(y)\,dy, \quad \psi(y) = \zeta(y) - y\zeta(1), \quad \xi(y) = \int_0^y \frac{\delta}{u^{-2}}\,dy\right)$$

2. Let us investigate the deformation of the velocity profile in a channel whose walls are ideally segmented electrodes for $x > 0$ and insulators for $x < 0$. The loads connecting the electrodes are chosen such that the condition $j_y = -(1-K)$ = const is fulfilled at the walls for $x > 0$. The condition $K = 1$ corresponds to a channel whose walls are noncunductive along its entire length. Let the magnetic field be of the form

$$b(x) = 0 \quad (x < 0), \qquad b(x) = 1 \quad (x > 0) \qquad (2.1)$$

Further, let the velocity profile be homogeneous ($u^- \equiv 1$) and let the Hall parameter be different from zero. We shall now determine the velocity profile (**) $u_1^+(y)$.

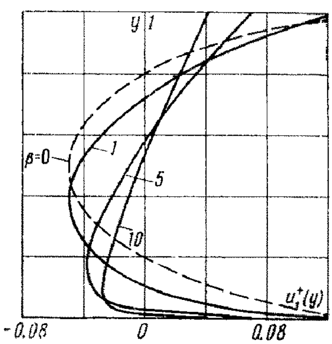

Fig. 2

From (1.10), (1.11) we obtain

$$u_1^+ = C + \int_0^y \gamma dy, \qquad C = -\int_0^1 \left(\int_0^y \gamma dy\right) dy$$

$$(\gamma = j_{x0}\,(0, y)) \qquad (2.2)$$

System (1.3) under the above conditions is solved in [8]. The expression for the current j_{x0} in the discontinuity cross section of the magnetic field is of the form

$$j_{x0} = \frac{1}{\beta} K \left[1 - \frac{2}{\sqrt{4+\beta^2}}\left(\cot\frac{\pi y}{2}\right)^{1-2k}\right]$$

$$\left(\pi k = \tan^{-1}\frac{2}{\beta}\right) \qquad (2.3)$$

The dependences $u_1^+(y)$ computed from Formulas (2.2), (2.3) for a channel with nonconductive walls ($K = 1$) for various β appear in Fig. 2. The broken curve

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*) By (1.11) u_1^+ is bounded and continuous only if $u^-(y)$ is a smooth function which does not vanish for $0 \leq y \leq 1$.

**) The asymptotic velocity profile is realized in practice for $x \gtrsim 1$.

corresponding to the flow of an isotropically conductive fluid is taken from [3]. The absolute value of the velocity perturbations diminishes as β increases. The point where $du_1^+/dy = 0$ moves closer to the lower wall. In a channel whose walls act as electrodes the velocity perturbations are, by (2.2), (2.3), smaller than those shown in Fig. 2 by the factor $1/K$.. The velocity u_1^+ at the walls (Fig. 2) is equal to 0.127, 0.0703, 0.0419 for β = 1, 5, 10, respectively.

3. Let us consider the flow of an isotropically conductive fluid in a channel with nonconductive walls when the unperturbed velocity profile is described by an arbitrary even smooth function $u^-(y)$. System (1.3) can be rewritten as

$$j_{x0} = -\frac{\partial\varphi_0}{\partial x}, \qquad j_{y0} = -\frac{\partial\varphi_0}{\partial y} - u^- b, \qquad \operatorname{div} \mathbf{j}_0 = 0 \tag{3.1}$$

$$\partial\varphi_0/\partial y = -u_w b \quad \text{for} \quad y = 0,\ y = 1$$

Here u_w^- is the velocity at the wall. For $u_w^- = 0$ system (3.1) is solved in [9]. Let $u_w^- \neq 0$. On conversion to the auxiliary variable $\Phi(x, y)$, system (3.1) becomes

$$\Delta\Phi = -b\frac{du^-}{dy} + u_w^- y \frac{d^2 b}{dx^2} \qquad (\varphi_0 = \Phi - b u_w^- y) \tag{3.2}$$

$$\partial\Phi/\partial y = 0 \quad \text{for} \quad y = 0,\ y = 1$$

Its solution is given by Formulas

$$\Phi = \sum_{\nu=1}^{\infty} \Phi_\nu(x) \cos 2r_\nu y + \frac{1}{2} b u_w^- \tag{3.3}$$

$$\frac{d^2\Phi_\nu}{dx^2} - 4r_\nu^2 \Phi_\nu = -b\mu_\nu - \frac{u_{10}}{r_\nu^2}\frac{d^2 b}{dx^2}, \qquad \Phi_\nu(\pm\infty) = \frac{\mu_\nu}{4r_\nu^2} b(\pm\infty) \tag{3.4}$$

$$\frac{du^-}{dy} = \sum_{\nu=1}^{\infty} \mu_\nu \cos 2r_\nu y, \quad \mu_\nu = 2\int_0^1 \frac{du^-}{dy} \cos 2r_\nu y \, dy$$

$$r_\nu = \frac{\pi}{2}(2\nu - 1), \qquad y = \frac{1}{2} - \sum_{\nu=1}^{\infty} \frac{\cos 2r_\nu y}{r_\nu^2}$$

The general solution of (3.4) can be readily represented in quadratures.

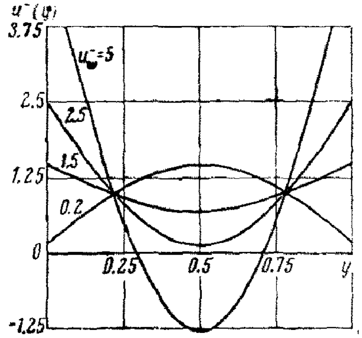

Fig. 3

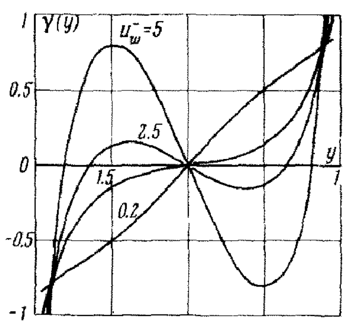

Fig. 4

If the function $b(x)$ is discontinuous (e.g. (2.1)) or piecewise-smooth, then Equation (3.4) must be solved in each segment where db/dx is continuous and the solutions matched on the basis of the continuity conditions for the quantities

$$\Phi_\nu + \frac{bu_w^-}{r_\nu^2}, \qquad \frac{d\Phi_\nu}{dx} + \frac{u_w^-}{r_\nu^2}\frac{db}{dx} \tag{3.5}$$

in passing through the discontinuity points (*).

The Joule dissipation Q in the channel and the pressure drop are

$$Q = \frac{\sigma h^2}{c^2} V^2 B^{*2} Q_0 \qquad \left(Q_0 = \int_{-\infty}^{+\infty}\int_0^1 (j_{x0}^2 + j_{y0}^2)\, dx\, dy\right)$$

$$Q_0 = u_w^- \sum_{\nu=1}^{\infty} \int_{-\infty}^{\infty} b\left(\frac{b\mu_\nu}{2r_\nu^2} - 2\Phi_\nu - \frac{\mu_\nu}{8r_\nu^4}\frac{d^2b}{dx^2}\right) dx - \sum_{\nu=1}^{\infty}\frac{\mu_\nu}{8r_\nu^2}\int_{-\infty}^{\infty} b\frac{d^2\Phi_\nu}{dx^2}\, dx \tag{3.6}$$

$$P_\infty = 2K^* - \sum_{\nu=1}^{\infty}\frac{1}{2r_\nu^2}\int_{-\infty}^{\infty} b\,(4r_\nu^2\Phi_\nu - b\mu_\nu)\, dx \tag{3.7}$$

The quantities Q_0 and P_∞ coincide in the case of an unperturbed velocity profile $(u_w^- \equiv 1,\ \mu_\nu = 0)$.

Let us consider flow in magnetic field (2.1). From (3.3) to (3.7) we find that

$$\Phi_\nu = \alpha_\nu \exp(2 r_\nu x) \qquad \text{for } x < 0$$

$$\Phi_\nu = -\alpha_\nu \exp(-2r_\nu x) + \frac{\mu_\nu}{4r_\nu^2} \qquad \text{for } x > 0 \qquad \left(\alpha_\nu = \frac{4u_w^- + \mu_\nu}{8r_\nu^2}\right) \tag{3.8}$$

$$j_{x0}(0, y) = \int_{-\infty}^{+\infty} j_{x0}\frac{db}{dx}\, dx = \gamma(y) = -\sum_{\nu=1}^{\infty} 2\alpha_\nu r_\nu \cos 2r_\nu y \tag{3.9}$$

$$j_y(-0, y) = u^-/2, \qquad j_y(+0, y) = -u^-/2$$

$$Q_0 = 2\sum_{\nu=1}^{\infty} r_\nu\alpha_\nu^2, \qquad P_\infty = \frac{1}{\zeta(1)}\int_0^1 \gamma(y)\,\psi(y)\, dy + \sum_{\nu=1}^{\infty}\frac{\alpha_\nu}{r_\nu} \tag{3.10}$$

Let us choose the following two families of unperturbed velocitiy profiles characterized by the same flow rate

$$u = u_w^- + 6(1 - u_w^-)\, y(1 - y) \tag{3.11}$$

$$u = u_w^- + \frac{\pi}{2}(1 - u_w^-)\sin \pi y \tag{3.12}$$

For family (3.11) we find that

$$\gamma(y) = -3(1 - u_w^-)\,\eta(y) - u_w^-\tau(y) \tag{3.13}$$

$$\left(\mu_\nu = \frac{12(1 - u_w^-)}{r_\nu^2},\ \tau(y) = -\frac{1}{\pi}\ln\tan\frac{\pi y}{2} = \sum_{\nu=1}^{\infty}\frac{\cos 2r_\nu y}{r_\nu},\ \eta(y) = R_3 - 4\int_0^y\int_0^y \tau\, dy\, dy\right)$$

$$Q_0 = \frac{u_w^{-2}}{2} R_3 + 3u_w^-(1 - u_w^-) R_5 + 4.5(1 - u_w^-)^2 R_7 \tag{3.14}$$

$$P_\infty = \frac{1}{\zeta(1)}\int_0^1 \gamma\psi\, dy + \frac{3}{2}(1 - u_w^-) R_5 + \frac{1}{2} u_w^- R_3 \tag{3.15}$$

$$\left(R_3 = \sum_{\nu=1}^{\infty}\frac{1}{r_\nu^3} = \frac{7}{25.79436},\ R_5 = \sum_{\nu=1}^{\infty}\frac{1}{r_\nu^5} = \frac{31}{295.1215},\ R_7 = \sum_{\nu=1}^{\infty}\frac{1}{r_\nu^7} = \frac{127}{2995.286}\right)$$

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*) Lines of discontinuity of the magnetic field replace narrow zones in which it varies rapidly. Conditions (3.5) follow from the continuity of the current j_{x0} and the quantity $\partial\varphi_0/\partial y$ in passing through the discontinuity cross sections [9].

For the family of profiles (3.12) we obtain (3.16)

$$\gamma(y) = -\frac{\pi}{4}(1-u_w^-)\cos\pi y - u_w^-\tau(y) \qquad \left(\mu_1 = \frac{\pi^2}{2}(1-u_w^-),\quad \mu_2=\mu_3=\ldots=0\right)$$

$$Q_0 = \frac{\pi(1-u_w^-)^2}{16} + \frac{u_w^-(1-u_w^-)}{\pi} + \frac{u_w^{-2}}{2}R_3 \tag{3.17}$$

$$P_\infty = \frac{1}{\zeta(1)}\int_0^1 \gamma\psi\, dy + \frac{1-u_w^-}{2\pi} + \frac{u_w^-}{2}R_3 \tag{3.18}$$

The function $\psi(y)$ and the quantity $\zeta(1)$ in Formulas (3.15) and (3.18) can be determined from the corresponding profiles of (3.11) and (3.12) in accordance with (1.11). Formulas (3.14) and (3.17) are valid for all $0 \le u_w^- < \infty$, while (3.15) and (3.18), as noted above, are valid only for u_w^- such that $u^- > 0$ over the entire interval (0, 1).

Let us consider the profiles of (3.12) (Fig. 3) in more detail. They are convex in the direction of the x-axis if $u_w^- < 1$ (i.e. if the velocity at the wall is smaller than the average velocity over the cross section), and concave if $u_w^- > 1$. For $u_w^- > \pi/(\pi-2) = 2.75$ there is a segment $y_1 < y < 1 - y_1$ ($y_1\pi = \sin^{-1}[2u_{\bar{w}}/\pi(u_w^- - 1)]$), on which $u^-(y) < 0$. When $u_w^- < 2.75$, $u^- > 0$ over the entire interval (0, 1). Let us investigate the function $\gamma(y) = j_{x0}(y)$ (Fig. 4). It is odd with respect to $y = \frac{1}{2}$. If $0 < u_{\bar{w}} < \pi^2/(\pi^2-4) = 1.68$, then $\gamma(y) < 0$ for $0 < y < \frac{1}{2}$. When $u_w^- > 1.68$, $\gamma(y) < 0$ for $0 < y < y_2$ and $\gamma(y) > 0$ for $y_2 < y < \frac{1}{2}$. Here $y = y_2$ is a root of Equation $\gamma(y) = 0$. It can be shown that $y_2 < y_1$ for the same $u_w^- > 2.75$. If $0 < u_{\bar{w}} < \pi^2/(4+\pi^2) = 0.712$, then the curves $\gamma(y)$ have an inflection point; if $u_w^- > 0.712$, they are convex upward. The character of the dependences $\gamma(y) = j_{x0}(0, y)$ and $u^-(y) = 2j_y(-0, y)$ make it possible to represent the electrical currents in the channel schematically. If $u_w^- < 1.68$, then there is one center of vortex currents at the point (0, $\frac{1}{2}$). For $1.68 < u_w^- < 2.75$ there are two new centers of vortex currents at the points (0, y_2) and (0, $1-y_2$) with the same direction of rotation as before. On the other hand, when $u_w^- > 2.75$, the two indicated cente... ...le

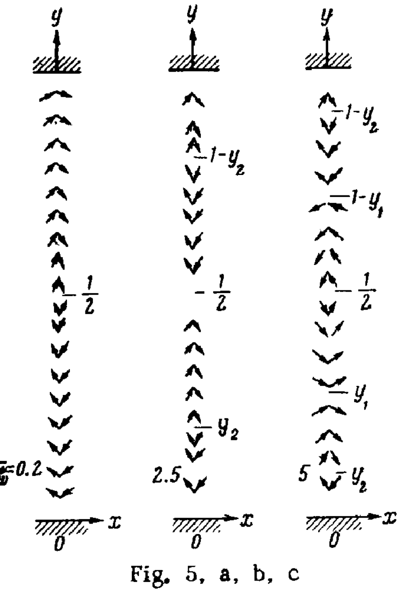

Fig. 5, a, b, c

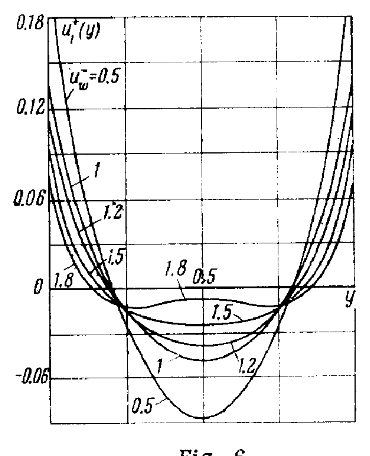

Fig. 6

closer to the walls, and closed current loops with opposite directions of rotation arise about the point $(0, \frac{1}{2})$. Fig. 5 shows the current directions along the line $x = 0$. We note that the discontinuity of the magnetic field disrupts the smoothness of the current lines at $x = 0$.

Fig. 6 shows the asymptotic velocity profiles. The velocity perturbation diminishes with increasing u_w^-. When the electrical current lines are of the character shown in Fig. 5a, the maximum pressure along the line $x = 0$ (by virtue of the force $(\mathbf{j} \times \mathbf{B})_{\parallel}$) occurs at the point $(o, \frac{1}{2})$. The velocity at the walls therefore increases while that near the axis diminishes. If the electrical current lines are as shown in Fig. 5b, the points of increased pressure (and hence of maximum slowing down of the fluid) shift away from the axis and closer to the walls. For sufficiently large u_w^- in the range $1.68 < u_w^- < 2.75$ the velocity $u_1^+(y)$ near the channel axis turns out to be positive.

The velocity correlation appears in Fig. 7 in the form of the function $K^*(u_w^-)$. It is evident that $2sK^*$ is equal (in the first approximation) to the difference between the momentum $\langle u^2 \rangle$ of the fluid in the cross sections $x \to \infty$ and $x \to -\infty$. As we know, given the same flow rate, a less (more) filled velocity profile is, as a rule, associated with a larger (smaller) momentum. It is evident from the curves of Fig. 6 that for $u_w^- < 1$ the velocity profile becomes more filled as a result of interaction between the fluid and magnetic field, so that $K^* < 0$. Fig. 7 also shows the dependence of the pressure loss P_∞ on the parameter u_w^-.

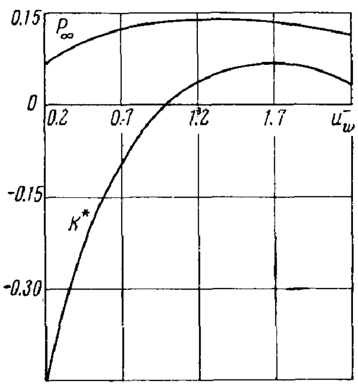

Fig. 7

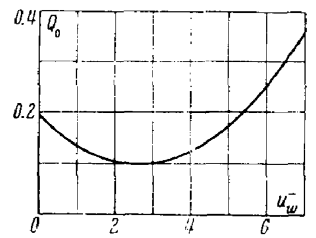

Fig. 8

If the correlation were not considered, P_∞ would appear to be a monotonously decreasing function. With allowance for the correlation, on the other hand, the function $P_\infty(u_w^-)$ is fundamentally different in character: it has a maximum.

Finally, Fig. 8 shows the function $Q_0(u_w^-)$. The Joule dissipation for a specified flow rate is determined chiefly by the velocity in the flow core. In accordance with Fig. 6, the absolute value of the velocity at the channel axis, first diminishes and then increases with increasing u_w^-. The function $Q_0(u_w^-)$ therefore has a minimum.

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